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CONJUGATE HEAT TRANSFER AND THERMOSTRESSED STATE OF A PLATE WITH RADIATION AND CONDUCTION HEATING

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The problem of combined analysis of the parameters of conjugate heat transfer and thermostressed state of a plate with its radiation and conduction heating in an absorbing-medium layer is considered. The effect of the optical characteristics of the medium on the parameters of conjugate heat transfer and the distribution of thermal stresses in a plate is studied.

The present paper considers conjugate heat transfer in a system consisting of a plane layer of an absorbing medium and a plate and the thermostressed state of the latter with unilateral heating of the plate by radiation and conduction. The heating source is modeled by an extended plane parallel to the plate. On this plane at $t > 0$ some limiting temperature T_1 is maintained; the plane of the plate opposite the medium has some temperature $T_0 = \text{const}$ ($T_0 < T_1$) and rests upon an absolutely solid base; the contact surface between medium and the plate is free. At the initial time instant, the entire system has temperature T_0. This formulation of the problem is typical in the analysis of temperatures and thermal stresses due to radiative heating of the structural elements of varions types of industrial furnaces [1]. We assume that the medium contacting the plate is gray and the surfaces of the emitter and the plate are gray and diffusely radiating and reflecting [2]. With allowance for the adopted assumptions and also assuming the thermophysical and thermomechanical properties of the medium and plate to be constant, we write the compound problem of conjugate heat transfer and thermoelasticity for a two-layer system (Fig. 1) in the following form

$$\frac{\mu}{\mathrm{Bu}}\frac{d\Psi}{dY} + \Psi = \Theta^4, \quad 0 < Y < 1; \tag{1}$$

$$\Psi^+(0) = \frac{\varepsilon_1\Theta_1^4 + 2\rho_1 E_3(\mathrm{Bu})\,\varepsilon_2\Theta_2^4 + 2\rho_1(\overline{A} + 2\rho_2 E_3(\mathrm{Bu})\,\overline{B})}{1 - 4\rho_1\rho_2 E_3^2(\mathrm{Bu})}; \tag{2}$$

$$\Psi^-(1) = \frac{\varepsilon_2\Theta_2^4 + 2\rho_2 E_3(\mathrm{Bu})\,\varepsilon_1\Theta_1^4 + 2\rho_2(\overline{B} + 2\rho_1 E_3(\mathrm{Bu})\,\overline{A})}{1 - 4\rho_1\rho_2 E_3^2(\mathrm{Bu})}; \tag{3}$$

where

$$\rho_1 = 1 - \varepsilon_1; \quad \rho_2 = 1 - \varepsilon_2;$$

$$\overline{A} = \int_0^{\mathrm{Bu}} \Theta^4 E_2(z)\,dz; \quad \overline{B} = \int_0^{\mathrm{Bu}} \Theta^4 E_2(\mathrm{Bu} - z)\,dz; \tag{4}$$

E_2, E_3 are integro-exponential functions [2];

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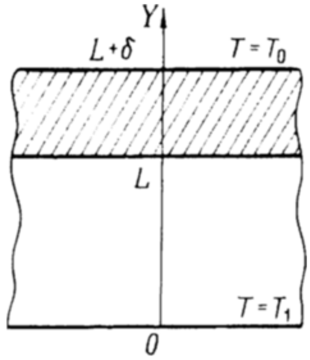


Fig. 1. Computational scheme.

$$\Theta_1 = \Theta|_{Y=0}\,; \quad \Theta_2 = \Theta|_{Y=1}\,;$$

$$\frac{\partial^2\Theta}{\partial Y^2} - 4\mathrm{Iw}\,\mathrm{Bu}\,(\Theta^4 - G) = \frac{1}{K_I}\,\frac{\partial\Theta}{\partial \mathrm{Fo_m}}, \quad 0 < Y < 1\,, \tag{5}$$

here $G = \frac{1}{2}\int\limits_{-1}^{1}\Psi d\mu$;

$$\frac{\partial^2\Theta_m}{\partial Y^2} = \frac{\partial\Theta_m}{\partial \mathrm{Fo_m}}, \quad 1 < Y < 2\,; \tag{6}$$

$$\Theta|_{Y=0} = 1\,; \tag{7}$$

$$\Theta|_{Y=2} = 0\,; \tag{8}$$

$$\Theta_m|_{Y=1} = \Theta|_{Y=1}\,;$$

$$-\left.\frac{\partial\Theta_m}{\partial Y}\right|_{Y=1} = K_{II}\left.\left(-\frac{\partial\Theta}{\partial Y} + 4\mathrm{Iw}Q_R\right)\right|_{Y=1}, \tag{9}$$

where

$$Q_R = \frac{1}{2}\int\limits_{-1}^{1}\Psi\mu d\mu\,; \tag{10}$$

$$\Theta|_{\mathrm{Fo_m}=0} = \Theta_m|_{\mathrm{Fo_m}=0} = 0\,; \tag{11}$$

$$\frac{d^2U}{dY^2} = \frac{1+\nu}{1-\nu}\,\frac{d\Theta_m}{dY}, \quad 1 < Y < 2\,; \tag{12}$$

$$\sigma_Y|_{Y=1} = 0\,; \tag{13}$$

$$U|_{Y=2} = 0\,. \tag{14}$$

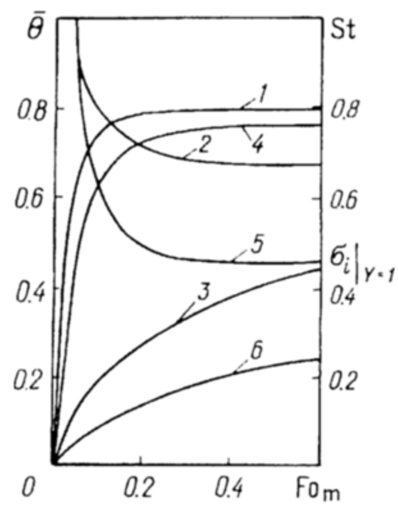


Fig. 2. Distribution of temperature $\overline{\Theta}$ (1), St number (2), and stress intensity on heated surface of plate (3) with time at Bu = 0.1; curves 4-6, for Bu = 1.

The dimensionless variables in Eqs. (1)-(14) are expressed by the relations

$$U = \frac{u_y}{\alpha_{\tau_m} (T_1 - T_0)\,\delta}, \quad \sigma_Y = \frac{\sigma_y}{\alpha_{\tau_m} (T_1 - T_0)\, E_m}; \quad \Theta = \frac{T - T_0}{T_1 - T_0};$$

$$\Theta_m = \frac{T_m - T_0}{T_1 - T_0}; \quad \Psi = \frac{I}{\sigma_0 (T_1 - T_0)^4/\pi}; \quad Y = \frac{y}{L}\,(0 \le y \le L);$$

$$Y = \frac{y + \delta - L}{\delta}\,(L \le y \le L + \delta); \quad \mathrm{Bu} = L\beta; \quad \mathrm{Iw} = \sigma_0 (T_1 - T_0)^3/\lambda;$$

$$\mathrm{Fo}_m = a_m t/\delta^2; \quad K_I = k_a k_\delta^2; \quad K_{II} = k_\lambda k_\delta,$$

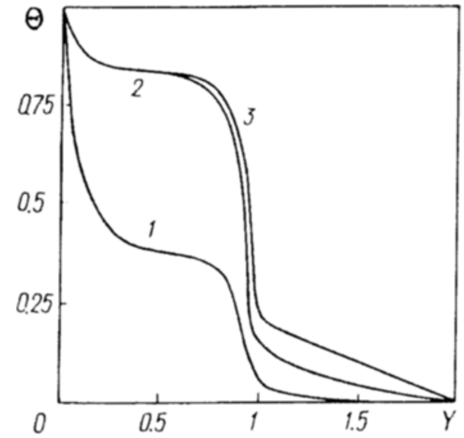
where

$$k_a = a/a_m; \quad k_\delta = \delta/L; \quad k_\lambda = \lambda/\lambda_m. \tag{15}$$

The subscript m in relations (15) indicates the plate characteristics; the characteristics of the absorbing-medium layer have no subscripts.

To solve problem (1) we used an implicit finite-difference scheme, though the algorithm for calculating a conjugate heat transfer problem has some specific features. Due to the nonlinear relation between the temperature and radiation intensity of the medium Eqs. (1) and (5) can be solved only by iterations. At the same time, temperature Θ_2 in condition (3) is not known a priori, since the problem is considered in a conjugate formulation. A computational experiment showed that the traditional approach to the solution of these problems [1] under certain conditions (when temperatures of the absorbing medium and conjugation surface become close) can lead to physically contradictory results. The cause of these errors is the fact that the conjugation conditions (9) in implementing the iteration scheme for the solutions [1] at each step are performed not simultaneously but successively.

In the present paper it is suggested that the conjugate heat transfer problem be solved using a through iteration scheme that satisfies the conditions of ideal thermal contact (9) at each iteration. For each time instant it can be described by the following sequence: 1) the problem of radiation transfer (1)-(3) is solved for a given temperature field [2, 3]; 2) initial coefficients of factorization are prescribed by condition (7) and then, based on a finite-difference approximation of Eq. (5), they are calculated for all the nodes in the layer; 3) the initial



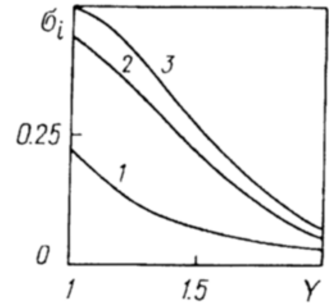


Fig. 3. Temperature distribution over the layer and plate thickness at Bu = 0.1: 1) $Fo_m = 0.02$; 2) 0.2; 3) 0.6.

Fig. 4. Distribution of stress intensity (Bu = 0.1) over plate thickness: 1) $Fo_m = 0.1$; 2) 0.6; 3) 1.2.

coefficients of factorization for the plate [4], which are then found by the difference analog of Eq. (6), are calculated based on conjugation conditions (9) and the obtained coefficients of factorization for the medium; 4) reverse factorization is performed using condition (8) and recursion formulas. The described sequence of steps is repeated until the specified accuracy is achieved, then a transition to the next time layer takes place.

Assuming that plate heating occurs at a relatively low rate, we calculate the thermal stresses for each time instant within the framework of a quasistatic problem of thermoelasticity [5].

When considering the problem of plate heating in a conjugate formulation, the question arises as to what calculation parameters will provide an integral estimate of the studied thermophysical process. To answer this question we represent the second of conjugation conditions (9) as follows:

$$-\left.\frac{\partial \Theta_m}{\partial Y}\right|_{Y=1} = \mathrm{St}\,(\overline{\Theta}^4 - \Theta_m^4|_{Y=1}), \tag{16}$$

where $\overline{\Theta} = \int_0^1 \Theta dY$ is the temperature of the medium averaged over the layer thickness; $\mathrm{St} = \sigma(T_1 - T_0)^3\delta/\lambda_m$ is the Stark number; σ is an unknown coefficient of radiation characterizing the intensity of plate heating in the considered system. It is obvious that solution of the problem in a conjugate formulation makes it possible to obtain, using relation (16), the St number by calculation. Multiple repetition of the problem solution allows one to derive dimensionless relations, which are presented in the form of tables:

$$\mathrm{St} = f_1\,(K_I\,,\ K_{II}\,,\ \mathrm{Iw}\,,\ \mathrm{Bu}\,,\ \mathrm{Fo_m})\,; \tag{17}$$

$$\overline{\Theta} = f_2\,(K_I\,,\ K_{II}\,,\ \mathrm{Iw}\,,\ \mathrm{Bu}\,,\ \mathrm{Fo_m})\,. \tag{18}$$

These relations allow caclulation of plate heating in a nonconjugate formulation, i.e., solution of heat-conduction Eq. (6) with the boundary conditions (8) and (16). The latter, in this case, replaces conjugation conditions (9). It is apparent that this approach is more convenient for study of the thermostressed state of the plate when it is necessary to know only its temperature field.

Figures 2-4 give some results of calculations ($K_I = 0.5$; $K_{II} = 0.01$; Iw = 25; $\varepsilon_1 = \varepsilon_2 = 1$). Figure 2 shows the effect of the optical thickness of the medium on the dynamics of its heating, intensity of heat transfer from the medium to the plate, and the stress-strain state of the latter. As Bu decreases, the mean temperature of the medium and also the St number are stabilized at higher levels. This is explained by the definition of the Bouger number itself. A decrease in Bu means a reduction of either the coefficient of absorption β or the geometrical thickness L of the medium. It is obvious that both factors facilitate heating of both the medium and the plate. This, in turn, affects the value of thermal stresses. An overview of the dynamics of heating of the studied system is given in Fig. 3. The rather rapid stabilization of the temperature profile in the medium is explained by the predominance of the radiative component in the mechanism of heat transfer (Iw = 25). A gradual stabilization of the distributions of thermal stresses over the plate thickness is also typical (Fig. 4). Here the maximum thermal stress $(\sigma_i|_{y=1})$ is determined by the temperature established on the conjugation surface when the process reaches the steady state.

The presented results indicate continuity of the temperature fields in the medium and plate. A reliable analysis of thermal stresses in the plate is impossible without consideration of the problem of heating in a conjugate formulation. The given scheme for transition from a conjugate formulation of the problem of plate heating to the solution of the proper heat conduction problem with given nonlinear boundary condition is a promising one and can be used in calculations of temperatures and thermal stresses in structures with other configurations.

NOTATIONS

u_y, displacement; σ_y, stress; λ, a, α_t and β, coefficients of thermal conductivity, thermal diffusivity, linear expansion, and absorption of medium; Θ, temperature; T_1, T_0, limiting temperature; I, intensity of radiation; σ_0, Stefan-Boltzmann constant; δ, L, thickness of plate and layer; Iw, Bu, and Fo, similarity criteria (radiative-conductive, Bouger, and Fourier); E, elastic modulus; ε_i, ρ_i $(i = 1, 2)$, emissivity factor and refractive capacities of surfaces with coordinates $Y = 0$ and $Y = 1$. Subscripts: m, metal; t, temperature.

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